B. Sc. Part-I

Paper - I

Theory of Relativity

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Time dilation :-

The first result of relativity is that time is not the same for all frames of reference. The simplest case for relativity is light traveling independently of the direction of motion.

To answer the question, we will describe a rather simple clock. The clock functions when a beam of light is emitted from a source/clock and travels to mirror. Upon striking the mirror, the light is reflected back, directly opposite the mirror and both observers A and B record the event ~~as~~ as figure below —

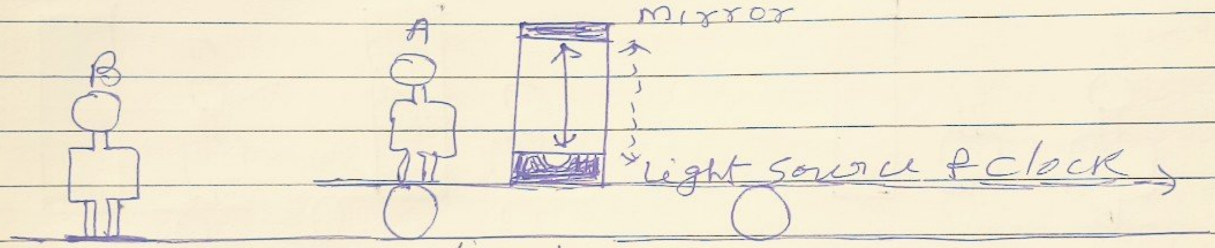


fig-1

From the point of view of observer A, the light goes straight up and down. While the ground and observer B are moving off to the left at velocity v.

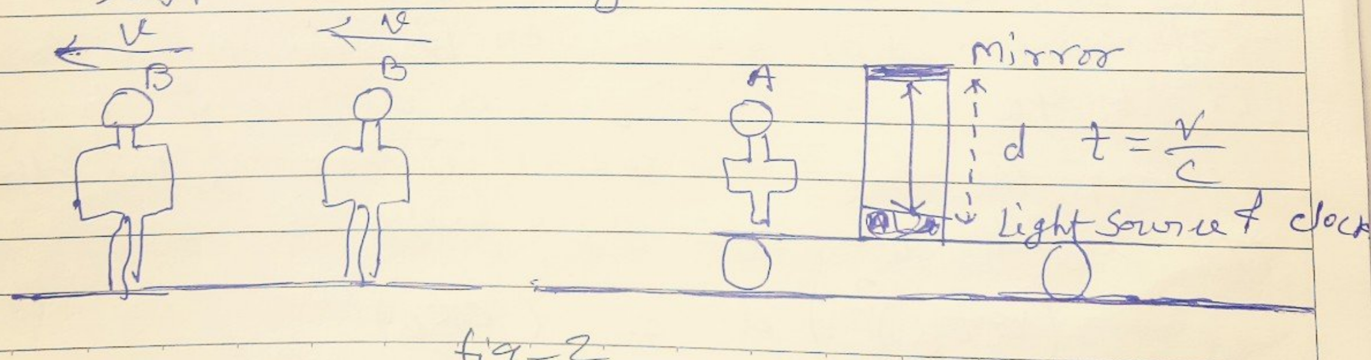


fig-2

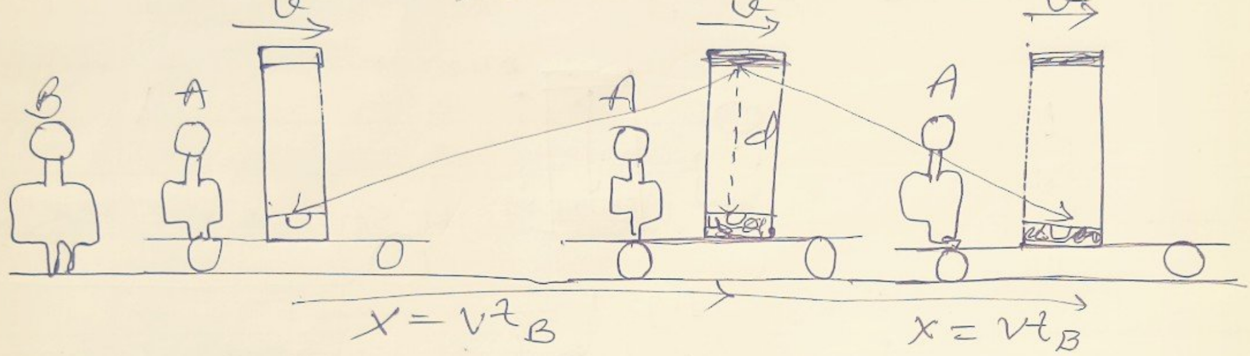
Teacher's Signature

If the distance between the mirror and the source is d, then the round-trip distance the light travels is 2d. Since the light travels with speed c, the one-way travel time t_A as measured by observer A is

$$t_A = \frac{d}{c}$$

The light arrives back at the clock in time $2t_A$.

What time interval will observer B see? Since the mirror moves with velocity v, to the right observer B will see the light move in a diagonal path as shown in figure below. From their point of view the light is travelling a longer distance, but it is still travelling at the same speed c, from postulate of relativity.



B's frame of Reference

Einstein's answer to the puzzle was that time works differently in this other frame of reference. For observer B, the time for the light to go one way is a different time t_B. Using the Pythagorean Theorem, the distance the light travels one way is

$$(vt_B)^2 + d^2 = (ct_B)^2$$

$$d^2 = (ct_B)^2 - (vt_B)^2$$
$$d = t_B\sqrt{c^2 - v^2}$$
$$t_B = \frac{d}{\sqrt{c^2 - v^2}}$$

This is the time to go one way from the source to the mirror. The return time is the same, so the total time is $2t_B$. We can relate this back to the time in A's frame of reference

$$t_B = \frac{t_A}{\sqrt{1 - v^2/c^2}}$$

Since the denominator is smaller than 1, the time interval measured by observer B will be longer than the time measured by observer A. We can, therefore, define time dilation as follows.

Clock moving relative to an observer run more slowly compared to the clocks that are at rest relative to the observer. This slowing down of time called time dilation.

For example, Earth-bound observers will measure time moving more slowly (than their own time) when observing a space ship moving with velocity v.

The situation is symmetric, with each inertial frame seeing the other's time as slower. We will shortly see, however, That the symmetry is not preserved if the space ship returns to earth.

~~we will see in this~~

Example — (A) The ship slows to a speed of 100,000 Km/h. Again, if the clock on board the ship measures 1.00 h. what is the amount of time measured on earth?

Sol.

Let us convert into 100000 Km/h into Km/s

A speed of $100000 \frac{Km}{h} = 10^5 \frac{Km}{h} = \left(10^5 \frac{Km}{h}\right)\left(\frac{1h}{3600s}\right)$

$= \underline{27.8} \text{ Km/s}$

The speed of light is $c = 300,000$ Km/s

Therefore, the speed of the space ship in terms of c is

$$\frac{27.8}{300000}c = 9.26 \times 10^{-5} c.$$

Also $1.00\ h = 3600 s$

Thus,
$$t = \frac{t_p}{\sqrt{1 - v^2/c^2}} = \frac{3600\ s}{\sqrt{1 - \frac{(9.26 \times 10^{-5})^2 \cancel{c^2}}{\cancel{c^2}}}}$$

$$= \frac{3600\ s}{\sqrt{1 - \frac{(9.26 \times 10^{-5})^2}{1}}} = \underline{3600.000015\ s}$$

This represents a difference in time between inertial frames of $1.5 \times 10^{-5} s$. which from on every day point of view is negligible. The fact that t depends upon the ratio v^2/c^2 show that when $v \ll c$, $\sqrt{1 - v^2/c^2} \cong 1$. Clearly the common speeds that we experience are smaller than 100000 Km/h, so there can be no equation that t and t_p can be considered essentially

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equal for day to day occurrences.

To date, the Speed of the ship given in Part B is about 40% greater than that achieved by any space ship or space Probe.

You may wonder if the effects of time dilation only apply to clocks. The answer is no! Clocks measure time differently because time itself is different in moving frames of reference. Everything happens slower in such circumstances. Biological systems, for instance are affected. The Process of aging the beating of one's heart, the Pace of breathing, are all subject to the effects of time dilation The heart rate of a person traveling in a space ship compared to what is measured on earth is slower. Recognition of this fact suggested what called today as the twin Paradox.